LA-10552-MS

UC-34B
Issued: September 1985

A Moon Base/Mars Base Transportation Depot

Paul W. Keaton

Los Alamos

Los Alamos National Laboratory
Los Alamos, New Mexico 875

A MOON BASE/MARS BASE TRANSPORTATION DEPOT

Paul W. Keaton

ABSTRACT

Placement of the next space outpost, after the low-Earth-orbit space station, will strongly affect the evolution of future space programs. The outpost will store rocket fuel and offer a haven to space workers, as well as provide a transportation depot for long missions. Ideally, it must be loosely bound to the Earth, easy to approach and leave, and available for launch at any time. One Lagrange equilibrium point, L_1(SE), between the Sun and the Earth and another, L_2(EM), in the Earth-Moon system have excellent physical characteristics for an outpost; L_1(SE), for example, requires less than 2% additional rocket propellant for docking there on the way to Moon bases or Mars bases.

We apply the rocket problem, the two-body problem, and the three-body problem in discussing alternative locations for space depots. We conclude that Lagrange point halo orbits are the standard by which other concepts for transportation depots must be gauged.

I. INTRODUCTION

An evolutionary manned space program will put outposts along routes to places with economic, scientific, and political importance. These outposts will be "filling stations" for storing rocket fuel, warehouses for holding bulk shielding material, assembly plants for building large structures, and transportation depots for connecting with flights to other destinations. Some outposts may produce oxygen and hydrogen from raw materials obtained elsewhere. Each outpost can provide a refuge from solar flare radiation, a hospital for emergencies, and an oasis to those whose missions call for prolonged space travel.

The obvious initial choice for such an outpost is a space station in low-Earth-orbit (LEO). LEO marks the first reasonable resting spot for climbing out of the deep potential well of the Earth's gravitational field and for leaving behind the aerodynamic drag of the Earth's atmosphere. Furthermore, LEO is still within the protection of the Earth's magnetic field so that galactic cosmic rays and lethal solar flares are not a life-threatening hazard to unshielded occupants. A LEO space station will also provide early opportunities to perfect life support systems and conduct physiological experiments. The knowledge gained will promote a better understanding of the problems of engaging people in long-duration space activities.

This first step, the low-Earth-orbit space station, is the largest. Placement of the second step will affect future space programs, including Moon bases, Mars bases, and manned access to Earth's geosynchronous orbit (GEO). The purpose of this paper is to discuss the physics of how to decide where that second outpost in space should be. An appendix is added to help readers who wish to extend the concepts developed herein.

II. LOCATING A TRANSPORTATION DEPOT

Any space habitat beyond the protection of the Earth's magnetic field will require some radiation shielding. The annual biological dose from galactic cosmic rays is about 50 rem (see Ref. 1); 5 rem per year has been allowed for radiation workers on Earth. In addition, several solar flares per 11-yr sun spot cycle would be lethal to astronauts without a radiation "storm cellar" of some type. Although the first few hours of a solar flare may be unidirectional, the radiation, which lasts for a day or two, soon becomes isotropic, so shielding is required on all sides. A transportation depot, therefore, should have easy access to some extraterrestrial source of bulk shielding material, such as lunar regolith. The cost of lifting inert material from the surface of the Earth would thus be avoided.

In general, we must consider where extraterrestrial resources will be obtained. If, for example, the main source is the Moon, it will be reasonable to have lunar manufacturing plants and remove only finished products from the surface. If much of the traffic is to and from the Moon, it may be sensible to have a transportation depot there also. However, there is strong evidence that Mars' two moons, Phobos and Deimos, have compositions similar to a carbonaceous chrondrite - - a type of meteorite that is rich in water and organics (see Ref. 2, p. 200). So we may find that the resources of the Moon, which are quite dry and contain only traces of carbon, and those of Mars' moons will complement the needs of a growing space program. Furthermore, there is a reasonable chance that one of the 73 catalogued Earth-crossing asteroids (see Ref. 3) could supply valuable materials. The same amount of rocket propellant is required to send unmanned freighters from LEO to the surface of Mars' outer moon, Deimos, as to the surface of the Moon, and about 10% less propellant is needed to reach asteroid 1982DB. This argues against placing a transportation depot on any body of substantial gravity, such as the Moon, because each trip to and from the body surface will extract an expenditure of rocket propellant at least equal to that needed to achieve escape velocity.

The many trips to and from a transportation depot will waste fuel and diminish its usefulness if it is poorly situated in space. Consider geosynchronous orbit (GEO), for example. Because of its operational significance, a space platform is needed at GEO. However, GEO is about the worst possible place for a transportation depot. More propellant mass is required to insert a rocket payload into circular geosynchronous orbit than to escape the Earth's gravitational field entirely (cf Ref. 4, p. 396). In addition, the return trip to the Earth from GEO requires more propellant mass than from nearly any other orbit radius (cf. Ref. 5, Chap. 5). Furthermore, the geosynchronous radius of 42 240 km (6.63 Earth radii) is at the outer edge of the Van Allen radiation belt and at the inner edge of the geomagnetic tail (cf. Ref. 6, p. 46). This may require considerable radiation shielding for the people staying there. Looking beyond GEO itself, and toward Moon bases, Mars bases, and products derived from extraterrestrial resources, we find no wisdom in placing a transportation depot at the Earth's geosynchronous orbit.

The velocity, v, of a transportation depot relative to its local gravitational center is also an important consideration for establishing its location in space. The faster an object is moving, the smaller will be the velocity increase, Δv, required to make a given kinetic energy increase. This is easy to see; in nonrelativistic mechanics, the kinetic energy, T, of a mass, m, is given by $mv^2/2$. For small increments of velocity, we may differentiate T with respect to v. Thus, the increase in kinetic energy is $\Delta T = (mv)\ \Delta v$, so that the larger v becomes, the smaller will be the Δv required to bring about a given ΔT. Since the larger the Δv the greater the rocket propellant mass required to accelerate the rocket, we can see the considerable savings in fuel if a rocket starts for Mars from LEO rather than from a much higher, slower orbit.

This last point leads to seemingly contradictory criteria for locating a transportation depot: it should not be tightly bound to a massive planet or moon because every encounter costs a lot of fuel and yet it should be capable of producing large velocities, which come from trajectories near massive bodies. One compromise is to establish highly elliptical orbits that give large velocities at perigee while requiring smaller binding energies to Earth than low, circular orbits. This compromise, which has many disadvantages (repeated passings through the Van Allen radiation belt, for example) will not be considered further here.

The ideal location for the second transportation depot, after the LEO space station, would be a spot that can be reached from LEO with no more than escape velocity, that requires no fuel to stay there, and that has an infinite launch window. Also, it would be easy to coast near the Earth from the ideal location; the rocket could, thereby, achieve a high velocity, leaving open options for igniting the engines to initiate interplanetary travel or aerodynamic maneuvering in the Earth's upper atmosphere. An added bonus would be obtained if the spot had velocity relative to the Moon so that lunar gravitational assists ("slingshots") could be used for increasing or decreasing a rocket's velocity. The ideal location for a transportation depot is depicted schematically in Fig. 1.

This logic leads us to discuss the subject of Lagrange points as candidates for locating transportation depots. If the Earth and Moon were fixed in space, there would be one point between them where

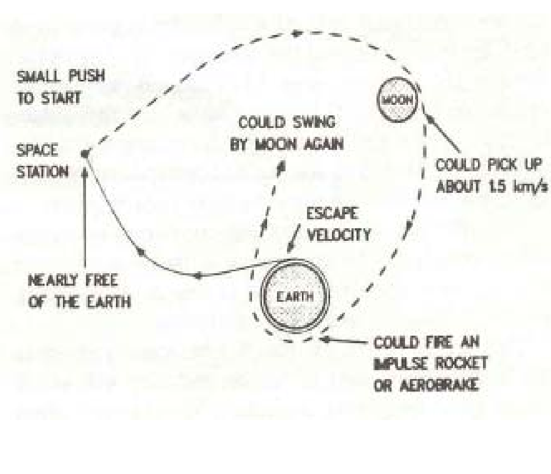

Fig. 1. Ideal place for a space station.

the attraction toward the Earth would just equal the attraction toward the Moon. Because the Earth and Moon are not fixed in space, but revolve around each other, there are, instead, five such equilibrium points-called Lagrange points, or libration points. These equilibrium points exist for other revolving two-body systems as well. The Earth-Moon Lagrange point, L_2(EM), and the Lagrange point between Sun and Earth, L_1(SE), are particularly favorable locations for a transportation depot. They can be reached from LEO with escape velocity, they are easy to approach and leave, and from them it is not difficult to swing close by the Earth to initiate a high-velocity Δv firing before going to other planets. They also afford easy access to the surface of the Moon. Their "halo" orbits can be maintained with almost negligible fuel year after year. This has been verified experimentally by the International Solar Earth Exploration (ISEE-3) satellite launched in 1978, which was maintained at L_1(SE) for 4 years with a station-keeping Δv expenditure of 10 m/s per year (see Ref. 7). In 1982, ISEE-3 was moved back to measure the Earth's geomagnetic tail and, in late 1983, with gravitational assists from the Moon, was moved on to rendezvous with the Giacobini-Zinner comet in September 1985. The name of the satellite has been changed to Intemational Comet Expedition (ICE). We will return to the subject of Lagrange points in discussing the three-body problem.

Figure 2 helps us put in perspective some of the important points of this paper: the amount of Δv required to reach LEO from the Earth's surface and the cumulative Δv necessary to reach GEO, the L_1(SE) Lagrange point, and other places. This is not a potential energy diagram, so Δv depends on the path taken. However, it is correct to imagine a rocket leaving the Lagrange point and picking up velocity as it "slides down" the curve to LEO, where it ignites its engines for a trip to Mars. Lunar gravitational assists can be included in the mission profile. Thus, the propellant needed to get to the Lagrange point is not wasted, and L_1(SE) can be thought of as the first stage of a multistage rocket trip from LEO to Mars.

In the rest of this paper, we deduce the findings presented in this section from the laws of classical mechanics.

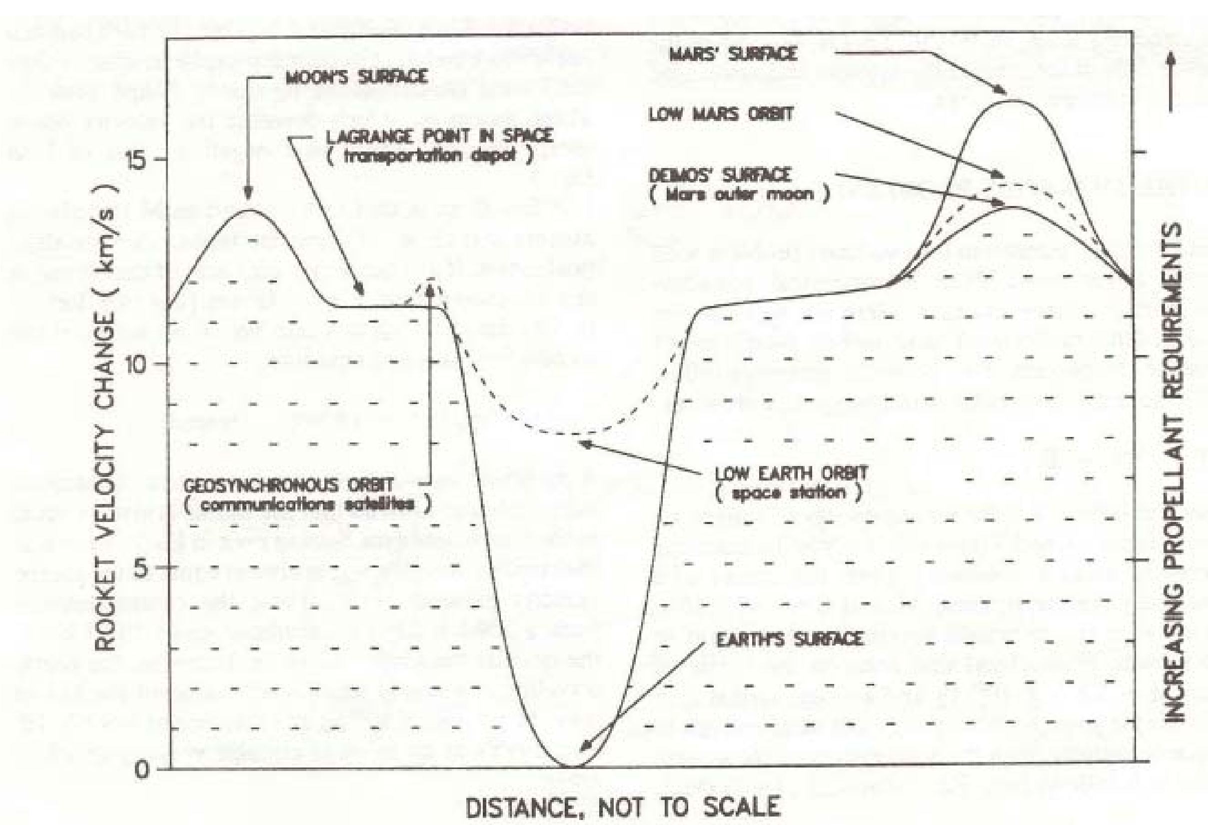

Fig. 2. Delta-v budgets for special locations

.III. THE ROCKET PROBLEM

The fundamental rocket equation equates the instantaneous change in momentum of a rocket of mass m and velocity v, with the instantaneous change in momentum of the exhaust propellant of velocity c relative to the rocket. It may be written

$$\text{THRUST} = m\dot{v} = -\dot{m}c \qquad (1)$$

where the dot indicates a derivative with respect to time. For constant c, Eq. 1 may be integrated exactly, and, if we remember that the initial rocket mass, m_i is the sum of the final rocket mass, m_f, and the propellant mass, m_p, Eq. 1 leads to

$$m_p \,/\, m_i = [1 - \exp(-\Delta v/c)] \ , \qquad (2)$$

which shows the propellant mass required to change the velocity of a rocket by a given amount. The exhaust velocity of the main shuttle engine, which burns hydrogen and oxygen, is about 4.5 km/s. To illustrate, a Δv of 3.9 km/s is required to place a communications satellite into circular geosynchronous orbit from a circular shuttle orbit of 250 km altitude. If the exhaust velocity is c = 4.5 km/s, then from Eq. 2, $m_p/m_i = 0.58$. That is, as it leaves the shuttle, 58 % of the mass of the satellite and associated rocketry is propellant mass.

IV. THE TWO-BODY PROBLEM

Historically, interest in the two-body problem with central forces arose from astronomical considerations of planetary motions. Here we will use the results of that earlier work to show how people might travel to the planets. Two powerful formulas follow easily from the conservation of energy, E, written as

$$T + V(r) = E \qquad (3)$$

where T is the kinetic energy of a mass, m, with velocity, v, and V(r) = - GMm/r is the potential energy of m at a distance r from the center of a spherically symmetric mass, M, and $G = 6.67 \times 10^{-11}$ $m^3/kg\text{-}s^2$ is the universal gravitational constant in MKS units. If an object that rests on the Earth (of mass $M = 5.976 \times 10^{24}$ kg and average radius $r_o = 6371$ km) is given a velocity, v_{esc}, just large enough to escape to infinity, then the total energy of the system is E = 0. It follows from Eq. 3 that $v_{esc}(r_0) = 2\ GM/r_0 = (11.19\ km/s)^2$. Generalizing, we may write the first fundamental equation as

$$v^2(r) = v_{esc}^2(r) + v_0^2 \quad \text{(unbound)}, \qquad (4)$$

where $v_{esc}^2(r) = 2$ GM/r, v_o is the velocity m would have after escaping from M (called the hyperbolic velocity), and v(r) is the velocity needed at r to achieve a hyperbolic velocity of v_o. In this case, with $E = mv^2/2 > 0$, m is said to be unbound and the trajectories trace hyperbolas, with the limiting case of E = 0 being a parabola. On the other hand, when a rocket of hyperbolic velocity v_o encounters a planet, the rocket velocity increases according to Eq. 4. If it does not collide or fire its engines, the rocket reaches its largest velocity closest to the planet surface (a point called periapsis) and then leaves the planet in a different direction, losing speed until it again reaches its former hyperbolic velocity. If instead, the rocket is to be captured into orbit around the planet, it can retrofire its engines at periapsis, slowing itself until the velocity is less than escape velocity, so that it cannot escape the planet. The change in velocity, Δv, will determine the high point (apoapsis) of the resulting orbit. If the planet has a sufficient atmosphere, as do Venus, Mars, Earth, and Jupiter, the periapsis can occur low enough to permit atmospheric drag to slow the rocket (that is, to aerobrake it) below escape velocity. These processes, which decrease the velocity below escape velocity, result in a negative value of E in Eq. 3.

If E < 0, m is said to be bound to M (we always assume that m << M), and the trajectories are elliptical orbits. If a is the semimajor axis of the ellipse, it can be shown that E = - GMm/(2a) (see Ref. 8, p. 79). Substituting this into Eq. 3, we arrive at the second fundamental equation,

$$v^2(r) = v_{esc}^2(r)\ [\ 1 - \ r \,/\, (2a)\ ] \qquad \text{(bound)}\ . \qquad (5)$$

A particular case of interest occurs when the satellite is in a circular orbit so that the radius is always equal to the semimajor axis. Setting r = a in Eq. 5 shows that the circular velocity, v_{cir}, is always equal to the escape velocity divided by $2^{1/2}$. Thus, the escape velocity from a 500-km LEO is calculated to be 10.77 km/s; the circular velocity, 7.62 km/s. Likewise, the Earth, traveling in a nearly circular orbit around the Sun of mass $M = 1.989 \times 10^{30}$ kg at a distance of 149.6×10^6 km, travels at an average circular velocity of 29.78 km/s.

The simplest example of orbital transfers from a circular radius of r_1 to a circular radius of r_2 can be worked out with Eq. 5. The so-called least-energy transfers, or Hohmann transfers, are obtained by directing the rocket thrust tangent to the orbit at r_1 so that the velocity is increased by Δv_1, just enough to coast on an elliptical path and reach its apoapsis at r_2 after traveling 180° around the dominant mass. Then a second tangential rocket thrust will increase the velocity by an amount Δv_2 to insert it into the new circular orbit at r_2. The major axis of the elliptical transfer orbit is $2a = r_1 + r_2$, which completely determines $v(r_2)$ in Eq. 5. Then $\Delta v_1 = v(r_1) - v_{cir}(r_1)$, $\Delta v_2 = v_{cir}(r_2) - v(r_2)$, and the total $\Delta v = \Delta v_1 + \Delta v_2$. Using Eqs. 4 and 5 repeatedly, we find that the Δv necessary for traveling from LEO to a rendezvous with Deimos is 5.5 km/s. For comparison, the Δv necessary to go to the Moon from LEO amounts roughly to the escape velocity from LEO, 3.2 km/s, plus the escape velocity from the Moon, 2.4 km/s, which total to 5.6 km/s.

Continuing the subject of Hohmann transfers, if we set $R = r_2/r_1$ and $S(R) = (\Delta v_1 + \Delta v_2)/v_{cir}(r_1)$, it follows from Eq. 5 and the above discussion that (see Ref. 5, p. 159)

$$S(R) = \left[\frac{2R}{1+R}\right]^{1/2} - \frac{1}{R^{1/2}}\left[1 - \left(\frac{2}{1+R}\right)^{1/2}\right]. \quad (6)$$

Figure 3 shows $\Delta v_1 + \Delta v_2$ for orbital transfers from a 500-km LEO. The various distances in Fig. 3 are given just to indicate a scale. Of course, the Sun's gravity would dominate orbits beyond the Earth's sphere of influence, which extends out from the Earth to about 930 000 km. The most interesting feature of S(R) is that it has a maximum, which is near R = 15.58. This means that more Δv, and hence more fuel, is required to place a payload from LEO into circular orbit at 100 000 km altitude than into a circular orbit at higher altitudes. Or, putting it differently, less Δv is required to go from LEO to infinity and back into the Moon's orbit around the Earth than to initiate a direct Hohmann transfer. Further details on this and bielliptic transfers are given in Ref. 5, p. 160.

A more realistic Hohmann orbit transfer will include changing the orientation of the plane by an angle $\Delta\theta$. The optimum maneuver executes a small angular change, $\Delta\theta_1$, at the lower orbit, and a larger angular change, $\Delta\theta_2$, at the higher orbit, so that the total angular change is $\Delta\theta = \Delta\theta_1 + \Delta\theta_2$. In that case, Eq. 6 generalizes to

$$\begin{aligned} S(R, \Delta\theta) = &\left[\frac{1+3R}{1+R} - \cos(\Delta\theta_1)\left(\frac{8R}{1+R}\right)^{1/2}\right]^{1/2} \\ &+ \frac{1}{R^{1/2}}\left[\frac{3+R}{1+R} - \cos(\Delta\theta - \Delta\theta_1)\left(\frac{8}{1+R}\right)^{1/2}\right]^{1/2} \end{aligned} \quad (7)$$

where again $R = r_2/r_1$ and $\Delta\theta_1$ is fixed so as to minimize $S(R, \Delta\theta)$. Figure 4 shows the total $\Delta v = \Delta v_1 + \Delta v_2$ necessary to transfer from a 500-km LEO to higher circular Earth orbits with a 28.5° plane change. The latitude of the Kennedy Space Center is approximately 28.5°, so that is a typical LEO to GEO transfer angle. Figure 4 also shows the necessary Δv_3 (dashed line) for returning to a 100-km perigee, from which aeromaneuvering is assumed feasible. An important feature of Δv_3 is that it reaches a broad maximum at 32 000-km altitude, which is close to the 36 000-km altitude of GEO. It therefore requires more propellant mass to return from GEO to the Earth than from almost any other circular orbit.

V. THE THREE-BODY PROBLEM

Analytic expressions for the orbits have not been found for the three-body problem, which is much more complicated than the two-body problem. Ultimately, trajectories are calculated by numerical methods on digital computers. For a clear and succinct elementary treatment of the three-body problem, the reader is referred to Chap. 62 of Ref. 9. Here we are concerned only with "halo orbits" around one of the Lagrange points.

Consider two masses, m_1 and m_2, which are a distance D apart and revolving around each other in perfect circles with an angular velocity, ω , under the influence of gravitational forces. It follows from the two-body problem that $\omega^2 = G(m_1 + m_2)/D^3$. The center of mass will be between m_1 and m_2, a distance α D from m_1, where $\alpha = m_2/(m_1 + m_2)$. We assume for definiteness that $m_1 > m_2$, so that $\alpha < 1/2$. We establish a right-handed coordinate system with its origin at the center of mass and rotating with an angular velocity, ω , such that m_1 and m_2 are always

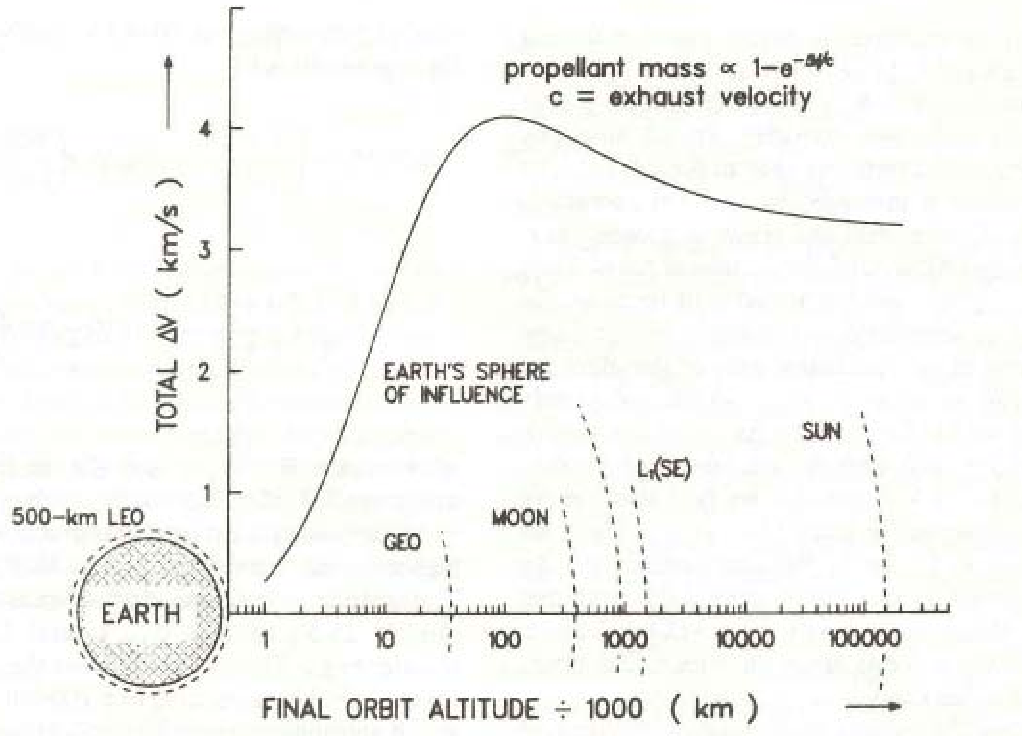

Fig. 3. Delta-v vs circular orbit altitude above the Earth.

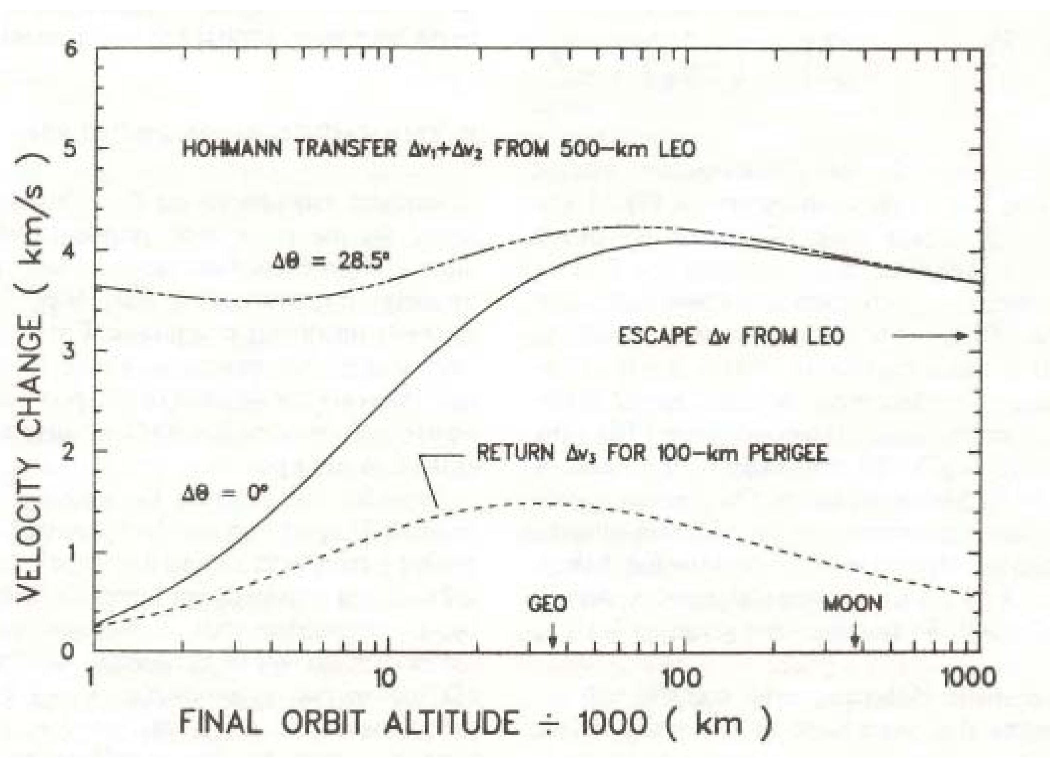

Fig. 4. Earth orbital transfer missions.

stationary on the x-axis. For example, designating coordinates as (x,y,z), we find the coordinates of m_1 are $(-\alpha D,0,0)$ and of m_2 are $((1-\alpha)D,0,0)$. We place the y-axis in the plane of rotation and the z-axis along the angular velocity vector. Now consider a third body of mass m that is so small it does not perturb the orbits of m_1 and m_2. These conditions describe the restricted three-body problem. Although it is specialized, this is an important problem because it describes reasonably well the situation of a rocket of mass m traveling in the Earth-Moon system or the Sun-Earth system.

The influence of m_1 and m_2 on m at some position (x,y,z) is described in the restricted three-body problem by a potential-like function,

$$U(x,y,z) = -\omega^2 D^2 \left[\frac{\left(x^2+y^2\right)}{2D^2} + \frac{1-\alpha}{\left(s_1/D\right)} + \frac{\alpha}{\left(s_2/D\right)} \right],$$
$$s_1 = \left[\left(x+\alpha D\right)^2 + y^2 + z^2 \right]^{1/2}, \text{ and} \qquad (8)$$
$$s_2 = \left[\left(x - \left(1-\alpha\right)D\right)^2 + y^2 + z^2 \right]^{1/2},$$

where s_1 is the distance between m and m_1 and s_2 is the distance between m and m_2. The equations of motion of m are (Ref. 9)

$$\begin{aligned} \ddot{x} - 2\omega\dot{y} &= -U_x, \\ \ddot{y} + 2\omega\dot{x} &= -U_y, \\ \ddot{z} &= -U_z, \text{ and} \\ \ddot{\vec{\mathbf{r}}} + 2\vec{\omega}\times\dot{\vec{\mathbf{r}}} &= -\,\mathbf{grad}\,U, \end{aligned} \qquad (9)$$

where $U_x = \partial U/\partial x$, etc., and the last of Eqs. 9 expresses the first three in vector notation. Because of the $\dot{x}$ and $\dot{y}$ terms in Eq. 9, U is not an ordinary potential function. In fact, particles can be trapped near maxima and saddle points of U as well as near minima. The Lagrange equilibrium points can be found by setting the gradient of U, which is the effective force on m, to zero. In doing so, the equilibrium points are seen to be in the plane of rotation, with z = 0. There are three collinear Lagrange points, L_1, L_2, and L_3, with y = 0, and two equilateral points, L_4 and L_5, with $s_1 = s_2 = D$. The five Lagrange points for the Earth-Moon system are shown in Fig. 5, along with L_1 and L_2 for the Sun-Earth system.

A three-body analogue to Eq. 5, which relates the velocity magnitude of m at a point in space to its current orbital parameters, may be deduced from Eq 9.

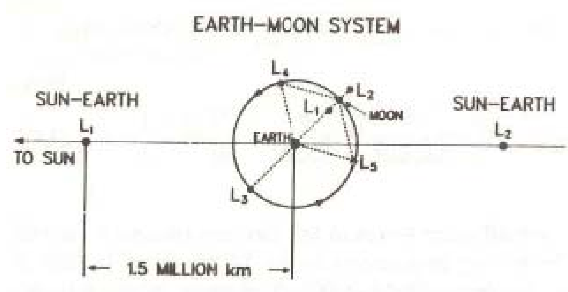

Fig. 5. Lagrange points near the Earth.

Taking the inner product of $\vec{\mathbf{v}} = \dot{\vec{\mathbf{r}}}$ and the last of Eqs. 9, we find that both sides of the resulting equation are perfect derivatives with respect to time, so their algebraic sum is a constant. Therefore,

$$\frac{1}{2}v^2 + U(\vec{\mathbf{r}}) = E = \text{constant}, \qquad (10)$$

expressing also a kind of energy conservation in the rotating system. To illustrate how Eq. 10 may be used, imagine that a satellite is known to have zero velocity when it is near the Sun-Earth L_1, which is located at $\mathbf{r_1} = (0.9900D,0,0)$. Substituting these coordinates into Eq. 10, we find that $E = U(\mathbf{r_1}) = -3.001 (\omega D)^2/2$. If then, with no thrust added, we later find that the satellite is at a 500-km perigee on the opposite side of the Earth from L_1 (this formula tells nothing about the trajectory necessary to arrive there), its potential must be $U = -3.131(\omega D)^2/2$, from which it follows that $v^2/2 = (3.131 - 3.001)(\omega D)^2/2$. That is, $v = (0.13)^{1/2}(\omega D)$. Because $(\omega D) = 29.79$ km/s, v = 10.74 km/s, which is very close to the escape velocity for LEO calculated from the two-body problem. This shows that the velocity necessary in LEO to reach L_1(SE) is between 10.74 km/s and 10.77 km/s.

Consider now the conditions necessary to establish a satellite around the Lagrange point L_1. At L_1, $(U_x)_1 = (U_y)_1 = (U_z)_1 = 0$ by definition, where the notation $(U_x)_1$ indicates $\partial U/\partial x$ evaluated at L_1, etc. Making a first-order Taylor expansion of the gradient of U about L_1 , setting $X = x - x_1$, and substituting into Eq. 9, we have the equations of motion near L_1:

$$\begin{aligned} \ddot{X} - 2\omega\dot{y} &= -\left(U_{xx}\right)_1 X, \\ \left(U_{xx}\right)_1 &= -\omega^2\left[1+2f^2\right]; \\ \ddot{y} + 2\omega\dot{X} &= -\left(U_{yy}\right)_1 y, \\ \left(U_{yy}\right)_1 &= -\omega^2\left[1-f^2\right]; \end{aligned} \qquad (11)$$

$$\ddot{z} = -\left(U_{zz}\right)_1 z\ ,\ \left(U_{zz}\right)_1 = +\,\omega^2 f^2,\ \text{and}$$

$$f^2 = \frac{1-\alpha}{\left(s_{1/D}\right)^3} + \frac{\alpha}{\left(s_{2/D}\right)^3}\ ,$$

where all other terms in the Taylor expansion vanish. The partial derivatives in Eqs. 11 are given in Ref. 9. To the first-order expansion, motion in the z direction is simple harmonic and independent of motion in the xy plane. Because L_1 is a mathematical saddle point, motion in the plane contains exponentially diverging solutions as well as periodic solutions, and therefore L_1 represents an unstable equilibrium. However, with the proper initial conditions, the diverging amplitudes can be set to zero (see Appendix), and bound periodic orbits, or halo orbits, result:

$$\begin{aligned} &X(t) = X_0 \cos(\,b\omega t)\ ,\\ &\quad b = \left[1 - f^2/2 + \left(f/2\right)\left(9f^2 - 8\right)^{1/2}\right]^{1/2};\\ &y(t) = -\left(X_0\gamma\right)\ \sin(\,b\omega t)\ ,\\ &\quad \gamma = \left(1 + b^2 + 2f^2\right)/\left(2b\right);\ \text{and}\\ &z(t) = z_0\ \cos(\,f\omega t)\ . \end{aligned} \qquad (12)$$

Note that the satellite is traveling on an elliptical path in a direction around L_1 opposite to that of the Earth around the Sun when the initial conditions are correct for a periodic orbit in the xy plane. The orbital parameters are $f^2 = 4.061$, $b = 2.086$, and $\gamma = 3.229$, which depend only on the ratio of m_2/ m_1, and $\omega = 2\pi$ rad/yr. Because $\gamma > 1$, the semimajor axis is always along the y-axis, perpendicular to the Sun-Earth line. Unlike elliptical orbits in the two-body problem, the period of revolution is independent of the size of the ellipse. The angular velocity in the plane is b ω; in the z direction, f ω. With L_1(SE), f = 2.02 and b = 2.09, so the halo orbit will have a period of about 6 months. These observations are all in agreement with the halo orbit of ISEE-3 described by Farquhar (cf. Ref. 7). For the Moon, L_1(EM) is located at x_1/D= 0.8369, where D = 384 400 km, and ω may be calculated by observing that the sidereal month is 27.32 days. From Eqs. 11 and 12, we see that f = 2.27, b = 2.33, and $\gamma = 3.59$. The period of rotation of a satellite around L_1(EM) is therefore about 12 days. Equations 11 and 12 are valid for any of the collinear Lagrange points, provided that proper values of s_1 and s_2 are substituted into Eq. 11 to find f^2. For example, x_2/D = 1.0100 and 1.1557 for L_2 of the Sun-Earth and Earth-Moon systems, respectively, so the corresponding values of f are 1.99 and 1.79. The period of a halo orbit around L_2(SE) is not significantly different from that of L_1(SE), but for L_2(EM), b = 1.86, and the period of the halo orbit is higher, about 15 days.

In exploring the basic physics of placing a space station at one of the Lagrange points, one last question remains. How large is the Δv push needed to make a rocket return from L_1(SE) to low-Earth-orbit? We attempt here only a heuristic estimate. Because L_1(SE) and the Earth are both revolving around the Sun with the same angular velocity, ω, and because they are different distances from the Sun, the Earth is traveling faster than L_1(SE) in a heliocentric inertial frame of reference. This amounts to a difference of about 0.01 x 29.78 km/s = 298 m/s, which can be taken as an initial estimate of the Δv necessary to eject from L_1(SE) and fall toward the Earth with essentially no angular momentum barrier. The number agrees well with the numerically-calculated value of 279 m/s given in Ref. 10. If, on the other hand, the rocket is orbiting L_1(SE) with a semimajor axis of about $X_0\,\gamma$ = 650 000 km, as was the ISEE-3, calculating $\dot{y}_{max}$ from Eq. 12 shows that the rocket reaches velocities relative to L_1(SE) as high as $(X_0\,\gamma\ b/D)\omega D$ = 0.009 x 29.79 km/s = 268 m/s. This occurs when the rocket is moving parallel with the Earth, so only an additional 298 - 268 = 30 m/s of Δv would appear to be needed to return to LEO from the halo orbit. This estimate is close to the ISEE-3 experiment that required Δv = 36.3 m/s for insertion into the halo orbit in November 1978. Subsequently, in June 1982, only Δv = 4.5 m/s was required to eject ISEE-3 from halo orbit and back into the geomagnetic tail. For our purposes here, we will define "very low Δv" to be less than 50 m/s.

A drawback of these very low Δv injections and ejections is that the transit times take months. The transit times can be reduced to weeks while still keeping Δv under 100 m/s. Perhaps the very low Δv encounters with L_1(SE) will be useful for only unmanned cargo ships carrying large masses. In addition, because the halo orbit period is 6 months, a space station will be at the proper place to receive freighters with very low Δv injections only twice a year. For each opportunity to dock, we estimate that the launch window will exist for about 3 weeks for very low Δv-class missions. These limitations will not pose serious problems when freight can be parked anywhere in halo orbit and boarded later by people at the appropriate time.

In addition to L_1(SE), other Lagrange points can be considered for a transportation depot. Although

L_2(SE) shares all of the same kinematic advantages, it lies in the Earth's geomagnetic tail and may not be suitable because of the radiation environment. Each of the five Earth-Moon Lagrange points could be a candidate for a transportation depot, but estimates of the injection velocities, such as those described above, indicate that the points require a Δv in the range of 1000 m/s. However, for L_2(EM) this can be overcome to a great extent by a clever maneuver. It has been shown that, using a retrograde lunar gravitational assist, the Δv necessary to enter a halo orbit at L_2(EM) is about 300 m/s instead of the 1230 m/s necessary for direct insertion from LEO (see Refs. 10 and 11). So we conclude there are two places in the Earth's vicinity well qualified for hosting a transportation depot-L_1(SE) and L_2(EM).

Looking beyond the Earth's vicinity, once the technology is developed to establish a manned space station around L_1(SE), we can use that technology at other places in the solar system. All of the major planets and moons have L_1 points that can sustain halo orbits. For example, Lagrange points of the Sun-Mars system have already been mentioned in connection with a Mars mission (see Ref. 12). An important factor in considering L_1(SM) at Mars is that the first manned missions there will carry return propellant, heavy shielding for radiation protection in transit, engines, and other things not needed on Mars' surface. The less tightly bound this equipment is to Mars, the less fuel will be needed to break it away later on the return trip. A ship going to Mars would save propellant by slowing at periapsis to just under escape velocity and coasting to L_1(SM). All of the equipment for returning to Earth would be placed in a halo orbit and left there during the descent to Mars. Later, reversing the procedure, the crew could return to Earth and waste very little fuel in the process. Eventually, a transportation depot at L_1(SM) would require its own space station, especially if water were found at Phobos or Deimos. But that is another subject. Here we are focusing on the second transportation depot, the one to follow the LEO space station.

Figure 6 emphasizes the "nodal" aspects of space stations at LEO and in a halo orbit around L_1(SE). It shows the Δv in kilometers per second required to move from each of the two nodes to GEO, Moon, L_2(EM), and Deimos. Notice that it costs very little extra total Δv to stop over at L_1(SE) on the way to one of these places from LEO. This makes L_1(SE) an important staging area for rocket propellant, water, bulk shielding material, and vehicle assembly for Moon and Mars expeditions. Freighters can go to L_1(SE) in months and take advantage of the very low Δv transfers into halo orbit. People can be carried there in a shorter time by paying a higher Δv expenditure in small orbital transfer vehicles (OTVs).

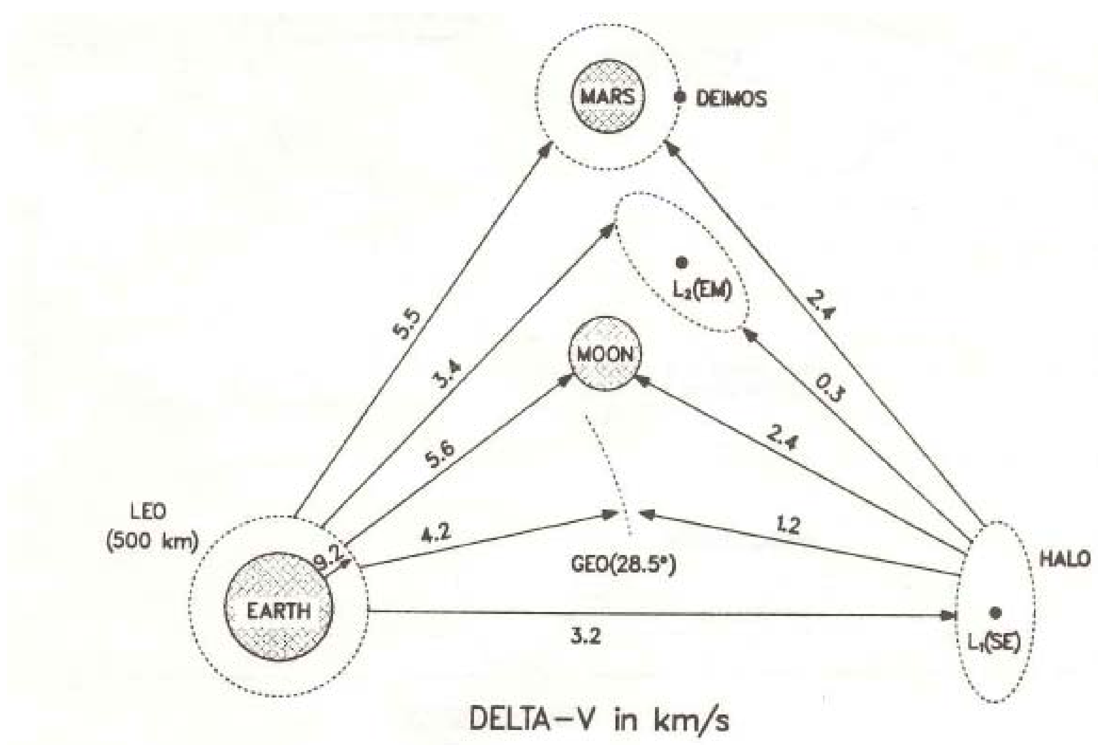

Fig. 6. Two space transportation nodes.

VI. SUMMARY

The key to a successful evolutionary space program is the placement of effective transportation nodes in the supporting infrastructure. Such outposts have always been important in opening frontiers. For the settlement of space, a Lagrange equilibrium point between the Sun and Earth has the nearly ideal physical characteristics of a transportation depot: it is very lightly bound in the Earth's gravitational well; it can be reached with essentially escape velocity; the launch window is always open; it can accommodate a wide range of plane angles for LEO space stations; its halo orbits require only 10 m/s per year of stationkeeping propellant to remain stable; and from there it is easy to leave and pass near the Earth at essentially escape velocity-affording several options. Rockets going to and from L_1(SE) can obtain free acceleration and braking by passing near the Moon's surface.

An in-depth study is necessary to determine the best place to put the next transportation depot after the LEO space station. But the laws of nature will not change. Lagrange point halo orbits are the present standard by which any alternative concept for a transportation depot must be gauged.

VII. ACKNOWLEDGMENTS

When the author was invited by the Lunar Base Steering Committee to express his views on this subject, he was reluctant to write a paper because many of the ideas are not original. However, he was persuaded by the argument that a tutorial format might provide background and establish the framework for future deliberations.

It is a pleasure to acknowledge several helpful conversations with Robert W. Farquhar, of NASA's Goddard Space Flight Center, during the course of this work.

APPENDIX

Many intricate problems in orbital mechanics can be solved with repeated applications of basic formulas, such as Eqs. 4 and 5, and the work often requires only a programmable calculator. Numerical examples are given in this Appendix, which is added to make a more nearly self-contained paper.

ROCKET ENGINE SPECIFIC IMPULSE

Rocket engines are characterized by the specific impulse, I_{sp}, which is the rocket thrust divided by the Earth's weight of propellant expelled per second. It follows from Eq. 1 that $I_{sp} = c/g$, where g (9.8 m/s^2 in MKS units) is the Earth's gravitational acceleration. The main shuttle engine, using hydrogen as a fuel and oxygen as an oxidizer, has a specific impulse of 460 s in vacuum. This is close to the theoretical maximum for chemical rocket engines.

NERVA, the thermal nuclear rocket engine developed in the 1960s, had a ground-tested specific impulse of $I_{sp} = 850$ s, with hydrogen as a propellant. The efficient way to store hydrogen and oxygen is in liquid form, which requires cryogenic storage tanks. When this is not practical, propellants are used that can be stored more easily. Solid and liquid propellants that can be stored at ordinary temperatures have specific impulses in the range of 300 ± 50 s.

ORBITAL TRANSITIONS

The most difficult transition is moving from the surface of the Earth to LEO. If there were no air resistance, it would require a Δv of 8.2 km/s to lift a rocket from Earth to a 500-km altitude LEO. Aerodynamic drag can be taken into account by adding about 1 km/s to that value. Thus, a very efficiently designed heavy lift launch vehicle (HLLV) requires $\Delta v = 9.2$ km/s to place a payload into LEO.

Once a rocket is in LEO, only Eqs. 4 and 5 are needed to find the necessary Δv for going to Deimos. The average distance of Mars from the Sun is r_2 = 1.525 times r_1, the average distance of Earth from the Sun. The major axis of a Hohmann transfer orbit from Earth to Mars is $2a = r_1 + r_2$. Using Eq. 5, we find that a rocket must have a velocity of 32.73 km/s at r_1 to reach r_2 at its aphelion. This calls for a hyperbolic velocity of $v_0 = 32.73 - 29.78 = 2.95$ km/s relative to the Earth. Knowing the required hyperbolic velocity to get to Mars, we can use Eq. 4 to show that a velocity of $[(10.77)^2 + (2.95)^2]^{1/2} = 11.17$ km/s is needed at a 500-km LEO to reach Mars. This calls for a velocity change of $\Delta v = 11.17 - 7.62 = 3.55$ km/s at LEO. If Mars and Earth were in the correct configuration at launch, namely Mars leading Earth by 44°, the rocket would coast for 259 days and reach Mars (see Ref. 4, Chap. 19). At that time, $r = r_2$ and $2a = r_1 + r_2$, so, according to Eq. 5, the rocket will be traveling at a velocity of 21.46 km/s. The average orbital velocity of Mars is 24.12 km/s, so Mars approaches the rocket with a hyperbolic velocity of $v_0 = 24.12 - 21.46 = 2.66$ km/s. The mass of Mars is 6.424×10^{23} kg, and Deimos travels in an approximately circular orbit 23 500 km from the center of Mars, with an orbital velocity of 1.35 km/s. If the rocket's trajectory is adjusted so that its periapsis is at 23 500 km, its velocity at that point will be, according to Eq. 4, $[2(1.35)^2 + (2.66)^2 1^{1/2} = 3.27$ km/s. The rocket must fire its engines a second time to change its velocity by $\Delta v = 3.27 - 1.35 = 1.92$ km/s. Therefore, a rocket can go from LEO to the orbit of Deimos around Mars with a total Δv of $3.55 + 1.92 = 5.47$ km/s.

The condition for minimizing $S(R,\Delta\theta)$ in Eq. 7 requires that

$$F(w) = R\sin(w)\left[\frac{3+R}{1+R} - \cos\left(\Delta\theta - w\right)\left(\frac{8}{1+R}\right)^{1/2}\right]^{1/2} - \sin\left(\Delta\theta - w\right)\left[\frac{1+3R}{1+R} - \cos\left(w\right)\left(\frac{8R}{1+R}\right)^{1/2}\right]^{1/2} \quad \text{(A-1)}$$

vanish, where $w = \Delta\theta_1$ is the root of Eq. A-1. This is easily found with a programmable calculator or computer routine for finding $F(w) = 0$ roots, by some procedure such as Newton's method. For LEO to GEO with $\Delta\theta = 28.5°$, $R = r_2/r_1 = 6.15$, and $\Delta\theta_1 = 2.26°$ and $\Delta\theta_2 = \Delta\theta - \Delta\theta_1 = 26.24°$.

FINDING LAGRANGE POINTS

The three collinear Lagrange points can be calculated by finding the real roots of $U_x = 0$, which, from

Eq. 8, amounts to finding the roots of

$$F(u) = u - \frac{(1-\alpha)(u+\alpha)}{|u+\alpha|^3} - \frac{\alpha(u-1+\alpha)}{|u-1+\alpha|^3} = 0, \qquad \text{(A-2)}$$

where u = x / D. To use a root-finding routine efficiently, initial estimates of the three roots for L_1, L_2, and L_3 are $u_1 = 1 - \alpha - (\alpha/3)^{1/3}$, $u_2 = 1 - \alpha + (\alpha/3)^{1/3}$, and $u_3 = -1 - 5\alpha/12$, respectively. For example, in the Earth-Moon system, $\alpha = 1.215 \times 10^{-2}$ and the initial estimate above yields $u_1 = 0.8285$, whereas the root is 0.8369. The other two Lagrange points, L_4 and L_5, are the points of an equilateral triangle, with $s_1 = s_2 =$ D. They may be calculated directly, where $x_4/D = x_5/D = (1/2 - \alpha)$, and $y_4/D = -y_5/D = 3^{1/2}/2$, where the x_j are the distances from the center of mass to L_j.

CHARACTERISTIC EQUATION

The coupled differential Eqs. 11 can be solved by substituting trial solutions, $X(t) = Ae^{\lambda t}$ and $y(t) = Be^{\lambda t}$, and forming a set of two simultaneous linear equations. A nontrivial solution for A and B will be found by setting the determinant of their coefficients equal to zero, yielding the characteristic equation:

$$\lambda^4 + (2 - f^2)\omega^2\lambda^2 + \omega^4(1 + f^2 - 2f^4) = 0 \qquad \text{(A-3)}$$

There are four solutions, $\lambda_1 = a\,\omega$, $\lambda_2 = -a\,\omega$, $\lambda_3 = i\,b\,\omega$, $\lambda_4 = -i\,b\,\omega$, and

$$X(t) = A_1e^{a\omega t} + A_2e^{-a\omega t} + A_3e^{i b\omega t} + A_4e^{-i b\omega t};$$

$$y(t) = B_1e^{a\omega t} + B_2e^{-a\omega t} + B_3e^{i b\omega t} + B_4e^{-i b\omega t};$$

$$z(t) = C_1e^{i f\omega t} + C_2e^{-i f\omega t}, \qquad \text{(A-4)}$$

where the A_j and B_j terms are linearly related. Let $B_1 = \beta A_1$, $B_2 = -\beta A_2$, $B_3 = i\gamma A_3$, and $B_4 = -i\gamma A_4$, we can cause the coefficients of $e^{a\omega t}$ and $e^{-a\omega t}$ to vanish with the initial conditions

$$\dot{X}(0) = (b\,\omega/\gamma)\,y(0), \qquad \dot{y}(0) = -(b\,\omega\,\gamma)\,X(0), \qquad \text{(A-5)}$$

where γ is given in Eq. 12. It follows that Eq. 11 produces bound halo orbits,

$$X(t) = X_0\cos(b\omega t) + (y_0/\gamma)\sin(b\omega t);$$

$$y(t) = -(X_0\gamma)\sin(b\omega t) = y_0\cos(b\omega t);$$

$$z(t) = z_0\cos(f\omega t - \phi_0), \qquad \text{(A-6)}$$

where, for simplicity, we have set $y_0 = \phi_0 = 0$ in this paper.

GRAVITATIONAL ASSISTS

We will calculate the maximum velocity that a rocket can receive from a given planet or moon, to identify the important parameters involved in gravitational assists (slingshots). If a satellite were stationary in some gravity-free inertial frame of reference, and if a massive body, M, came hurling by without colliding, it is easy to visualize that, after M has passed, the satellite would have some finite velocity in its original frame of reference. Its motion would be perturbed by the gravitational field of M. To generalize, if the satellite's original velocity were $\mathbf{v_s}$, its final velocity would be $\mathbf{v_s}'$, which may differ from $\mathbf{v_s}$ in both direction and magnitude. The change in kinetic energy per unit mass would be

$$\Delta E = \tfrac{1}{2}(v_s'^2 - v_s^2) \qquad \text{(A-7)}$$

But if this same encounter were seen by an observer on the planet, the satellite would be hurling toward the planet in a hyperbolic orbit with a velocity v_0, and, if there were no collision, it would arrive at some minimum distance, r_{min} from the center of the planet (periapsis). By then it would have moved through some angle δ, where geometric considerations show that $\sin(\delta) = 1/[1 + 2v_0^2/v_{esc}^2(r_{min})]$, and it would recede at a new angle, 2 δ, from its original direction and return to its original velocity, v_0.

Thus, the problem of calculating gravitational assists is made easier by transforming from the original inertial frame into the planet's rest frame (where it is simpler), and then back into the original inertial frame. This transformation may be used to find $\mathbf{v_0}$ and $\mathbf{v_0}'$, where

$$|\mathbf{v}_0| = |\mathbf{v}_0'|, \qquad \text{(A-8)}$$

but their directions differ by the angle 2δ.The velocities are related to each other and to ΔE by

$$\mathbf{v_s} = \mathbf{v_p} + \mathbf{v}_0 \ ; \quad \mathbf{v_s}' = \mathbf{v_p} + \mathbf{v}_0' \ ;$$

$$\Delta E = \frac{1}{2}\left(\mathbf{v_s}'^2 - \mathbf{v_s}^2\right)$$

$$= \frac{1}{2}\left(\mathbf{v_s}' + \mathbf{v_s}\right) \bullet \left(\mathbf{v_s}' - \mathbf{v_s}\right) \qquad \text{(A-9)}$$

$$= \mathbf{v_p} \bullet \left(\mathbf{v}_0' - \mathbf{v}_0\right);$$

$$\Delta E = 2\ v_0 v_p \sin(\delta)\cos(\psi)\ ,$$

where $\mathbf{v_p}$ is the planet's velocity in the inertial frame, ψ is the angle between $\mathbf{v_p}$ and ($\mathbf{v_0}' - \mathbf{v_0}$), and where we have noted that | $\mathbf{v_0}' - \mathbf{v_0}$ | = $2v_0 \sin(\delta)$. The maximum for ΔE will be found when ψ = 0, and substituting for sin(δ), the maximum energy change is

$$\Delta E_{max} = \frac{2\ v_p v_0}{\left[1 + \dfrac{2\ v_0^{\,2}}{v_{esc}^{\,2}(r_{min})}\right]}\ , \qquad \text{(A-10)}$$

where $v_0 = [v_s^2 + v_p^2 - 2\ v_s v_p \cos(\theta_{sp})]^{1/2}$, and θ_{sp} is the angle between $\mathbf{v_s}$ and $\mathbf{v_p}$. The higher the escape velocity, the greater the ΔE_{max}, so we must make r_{min} as small as possible, the limits being, of course, the radius of the planet and the height of its atmosphere. Equation A-10 has a peak value when $v_0 = v_{esc}(r_{min})/2^{1/2}$, so

$$\Delta E_{max}(\text{peak}) = \frac{v_p\ v_{esc}(r_{min})}{2^{1/2}} \qquad \text{(A-11)}$$

For the Moon in a geocentric inertial frame, $v_p = 1.02$ km/s and $v_{esc} = 2.3$ km/s at a 100-km altitude, so $E_{max}(\text{peak}) = (1.83)^2 / 2\ \text{km}^2/\text{s}^2$. That is, the velocity of a satellite or rocket can be increased in a lunar gravitational assist by as much as 1.83 km/s, providing the v_s and θ_{sp} are adjusted properly. Notice that, in general, as the satellite velocity becomes much larger than the escape velocity, the ΔE_{max} becomes small.

REFERENCES

1. R. Silberberg, C. H. Tsao, J. H. Adams, Jr., and John R. Letaw, "Radiation Transport of Cosmic Ray Nuclei in Lunar Material and Radiation Doses," in *Proceedings of the Lunar Bases and Space Activities of the 21 Century Symposium*, October 29-31, 1984, to be published by the Lunar and Planetary Institute, Houston, Texas.

2. Michael H. Carr, *The Surface of Mars* (Yale University Press, New Haven, Connecticut, 1981).

3. C. O. Lau and N. D. Hulkower, "On the Accessibility of Near-Earth Asteroids," AAS/AIAA Astrodynamics Specialist Conference, August 12-15, 1985, Vail, Colorado, AAS 85-352.

4. J. W. Cornelisse, H.F.R. Scheyer, and K. F. Wakker, "Rocket Propulsion and Space Flight Dynamics" (Pitman Publishing Corp., New York, 1979).

5. Laurence G. Taff, *Celestial Mechanics* (John Wiley and Sons, New York, 1985).

6. John T. Gosling, Daniel N. Baker, and Edward W. Hones, Jr., "Journeys of a Spacecraft," *Los Alamos Science*, No. 10, published by the Los Alamos National Laboratory (Spring 1984).

7. Robert Farquhar, Daniel Muhonen, and Leonard C. Church, "Trajectories and Orbital Maneuvers for the ISEE-3/ICE Comet Mission," AIAA/AAS Astrodynamics Conference, August 20-22, 1984, Seattle, Washington.

8. Herbert Goldstein, *Classical Mechanics*, 1st ed. (Addison-Wesley Publishing Co., Reading, Maine, 1950).

9. Edward A. Deslodge, *Classical Mechanics* (John Wiley and Sons, New York, 1982).

10. Robert Farquhar and David Dunham, "Libration-Point Staging Concepts for Earth-Mars Transportation," contributed paper in "Proceedings of the Manned Mars Mission Workshop," Marshall Space Flight Center, Huntsville, Alabama, June 10-14, 1985 (to be published).

11. Robert W. Farquhar, "A Halo-Orbit Lunar Station," *Astronautics and Aeronautics Journal* (June 1972).

12. Robert W. Farquhar, "Future Missions for Libration-Point Satellites," *Astronautics and Aeronautics Journal* (May 1969).